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Capillary Rise and Infiltration in Sand – Phenomena, 1D Tests and Analysis

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Kapillaraufstieg und Infiltration in Sand – Phänomene, 1D Versuche und Analyse

Zusammenfassung

Ergebnisse von 1D-Laborversuchen zur Infiltration und zum Kapillaraufstieg in anfänglich trockenem Sand werden vorgestellt. Für die ersten Untersuchungen zu den beiden Phänomenen wurden bewusst sehr einfache Bedingungen gewählt: Ein möglichst einfacher Versuchsaufbau und ein einziger, sehr gleichförmiger, Bodentyp sind Gegenstand der Untersuchungen. Die Dauer der beiden Prozesse wurde durch Messung des Fortschreitens der Benetzungsfront bestimmt. Diese Trennlinie zwischen dem nassen und dem trockenen Bereich bewegt sich bei der Kapillaraufstiegsbewegung von unten nach oben und bei der Infiltration von oben nach unten. In dem sehr gleichmäßigen Sand ist in beiden Fällen die Benetzungsfront nahezu eben. Im Allgemeinen hängt die Beschaffenheit der Benetzungsfront (eben, wellig, mit Fingern) von Faktoren wie Gleichförmigkeit oder Homogenität, Größe und Verteilung der Bodenpartikel, Eindringgeschwindigkeit und anfänglichem Wassergehalt des Bodens ab. Je nach Lagerungsdichte erfolgt die Eindringung in die 1 m hohe Sandsäule in einem Zeitraum von etwa 20 Minuten. Die endgültige Kapillaranstiegshöhe, die nach etwa zwei Stunden erreicht wird, beträgt etwa ein Drittel der Säulenhöhe. Die resultierenden Raten werden mit Ergebnissen aus der Literatur verglichen. Auf diese Weise lassen sich bestimmte Skalierungsabschnitte für den Kapillaranstieg bestimmen. Während des Prozesses ändert sich die Zeitabhängigkeit beim Kapillaranstieg: Zeitabschnitte, die von unterschiedlichen Zeit-Exponenten abhängen, können identifiziert und unterschiedlichen Mechanismen zugeordnet werden.

Stichwörter:

Kapillaraufstieg, Infiltration, poröse Medien, Benetzungsfront, teilgesättigte Böden

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Abstract

Results of 1D infiltration and capillary rise laboratory tests in initially dry fine sand are shown. To start investigations on the two phenomena, deliberately, basic conditions have been chosen: a simple experimental setup and a single, very uniform, soil type are subject of the investigations. The duration of the two processes has been observed by measuring the progress of the wetting front. This separating line between the wet and the dry region moves from the bottom to the top for capillary rise and from the top to the bottom for infiltration. In the very uniform sand in both cases the wetting front is almost plane. In general, the formation of the wetting front (plane, wavy, with fingers) depends on factors like uniformity or homogeneity, sizes and distribution of soil particles, infiltration rate and initial water contents of the soil. Depending on the packing density infiltration into the sand column of 1m takes place in approximately 20 minutes. The final capillary rise height which is reached after around two hours is about one third of the column height. The resulting rates are compared to curves from literature. In this way, certain scaling regimes of capillary rise can be determined. During the process the time dependence of capillary rise changes: periods depending on different exponents of time can be identified and attributed to different mechanisms.

Keywords:

Capillary rise, infiltration, porous media, wetting front, unsaturated soil

Capillary Rise and Infiltration in Sand – Phenomena, 1D Tests and Analysis

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Abstract

Results of 1D infiltration and capillary rise laboratory tests in initially dry fine sand are shown. To start investigations on the two phenomena, deliberately, basic conditions have been chosen: a simple experimental setup and a single, very uniform, soil type are subject of the investigations. The duration of the two processes has been observed by measuring the progress of the wetting front. This separating line between the wet and the dry region moves from the bottom to the top for capillary rise and from the top to the bottom for infiltration. In the very uniform sand in both cases the wetting front is almost plane. In general, the formation of the wetting front (plane, wavy, with fingers) depends on factors like uniformity or homogeneity, sizes and distribution of soil particles, infiltration rate and initial water contents of the soil. Depending on the packing density infiltration into the sand column of 1m takes place in approximately 20 minutes. The final capillary rise height which is reached after around two hours is about one third of the column height. The resulting rates are compared to curves from literature. In this way, certain scaling regimes of capillary rise can be determined. During the process the time dependence of capillary rise changes: periods depending on different exponents of time can be identified and attributed to different mechanisms.

1 Introduction

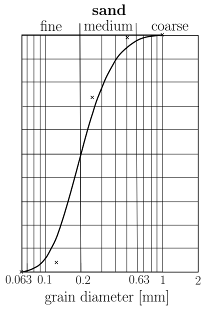
Starting point of the considerations in this article are observations concerning the watering of pot plants. One point is that – at least for dry, compacted material – water given to a plant initially builds a fluid layer above the potting soil before it penetrates into the pore space amongst the solid material of the potting soil. The reason has to be sought mainly in the properties of the potting soil. The soil type (cohesionless or cohesive soil, organic soil) determines both the particle/void-space size and the grain shape (rather spherical, platelet-shaped or fibrous), i.e. the geometry of the sample. Porosity and tortuosity – the latter implying the complexity of the pore system – are further factors specifying the permeability of the medium. To answer the question how fast water enters a soil certainly also the initial water content is decisive. If the soil is dry at the beginning of the process it behaves differently to initially wet material. The time before the water starts to penetrate into the pore space is much longer for dry than for wet material. The reason lies in the different capillary pressures for the same moisture content depending on the initial filling of the pores. If a sample is initially water saturated and then drained (process of drainage) a drying curve results. On the other hand, if the sample is initially dry and water is supplied until saturation is reached (process of imbibition) a wetting curve is the consequence. These two curves build a hysteresis loop, i.e. for one degree of saturation not only one value of the capillary pressure is possible (see e.g. [6][1]). Generally, the capillary pressure strongly depends on the degree of saturation (the volume of water in the voids divided by the the volume of the whole pore space which is filled by the two immiscible fluids water and air). Thus, besides the soil type, also types and amounts (or respectively, the saturations) of the pore fluids are of importance (see e.g. [3]) for the penetration behavior of the liquid into the soil. Also the viscosity of the material and thereby the temperature have a certain influence. The watering of plants in a pot surely is also effected by the degree of compaction and soil water repellency (see e.g. [28]). Also the degree of root penetration is relevant. Together with the water also nutrients or pollutants may be transported and possibly attach to the solid and influence the flowability (physical adsorption processes in soils have been investigated e.g. in [2]).

Besides the physical properties of the materials an important point is, whether the plant is watered from above or from below (the latter if you put the water into a flower pot coaster from which it can rise into the soil). In the first case, the water gets into the soil by infiltration, in the second case, capillary action is needed so that the water will rise. Capillary rise, action or motion – as the capillary effect is also called – occurs because of inter-molecular forces between the liquid(s) and surrounding solid surfaces. It strongly depends on the geometry, especially on the size of the channels or pore spaces in which it takes place. In soils, therefore, it depends on the soil type and thus on the particle sizes, the size of the pore space, the permeability of the material, the particle form – just overall on the geometry of the material (see e.g. [5] or [17]).

The first paragraphs emphasize that the topic of this paper is relevant to different target groups: geophysical and environmental engineers and those working in agriculture primarily are interested in the environmental consequences of processes of the water cycle. Thus, they analyze the impacts of e.g. heat, droughts and heavy rains. One question is, for example, the influence of the initial water contents for the penetration behavior of the water. If it cannot penetrate into the soil immediately, a water table on the ground develops, the water partly runs off at the surface and penetrates only slowly into the soil (so called ponding infiltration). Of course, this is important for the cultivation of plants and harvest success. The amount of rain also controls the need of irrigation which is a form of controlled infiltration into the subsoil. There is extensive literature focusing on environmental aspects, see e.g. [16], [38], [43]. Another group of researchers studying mainly the flow of the fluids are hydrologists. Various – often empirical – approaches to describe the flow processes during infiltration and capillary rise have been developed, see e.g. [11], [14], [19], [27], [33], [51], [59]. Geotechnicians and researchers dealing with soil mechanics are mainly interested in the influence of several soil types which entail different geometries, permeabilities, capillary pressures etc. (for an overview see [62]). Many publications on theoretical aspects, experimental verification and measurement methods appeared, see e.g. [9], [29], [42], [45], [49], [68]. Also the interaction of the constituents of soils and the physical behavior of solids and fluids which can be described by one- or multi-component models are in the focus of their work, see e.g. [1], [3]. Several studies also deal with theoretical aspects of the problems. Mathematicians and researchers from mechanics and thermodynamics answer questions about hysteresis (which appears in the

[1]The author married in 2016 and changed her name from Albers to Detmann.

Table 1: Physical properties of the fine sand used in the laboratory tests.



effective grain size $d_{10} = 0.1125$ mm		
grain density $\rho_S = 2.6466$ g/cm^3		
coefficient of uniformity $C_u \approx 2 \rightarrow$ very uniform		
hydraulic conductivity $K = 1.86 \cdot 10^{-4}$ m/s		
	dense packing	loose packing
porosity	$n = 0.384$	$n = 0.424$
dry density	$\rho_d = 1.6304$ g/cm^3	$\rho_d = 1.5251$ g/cm^3
relative density	$D = 95.9$ %	$D = 58.1$ %

(a) (b) (c)

Figure 1: (a) Schematic representation of the setup of two simple one-dimensional tests on an initially dry fine-sand sample. (b) and (c) experimental setup for capillary rise and infiltration, photos: Riemenschneider [65].

capillary pressure-saturation relation but also in plasticity), the stability of the wetting front, numerical analysis etc., see e.g. [7], [12], [35], [36], [40], [56], [57], [61], [71], [76]. Some of these works are picked up and studied in the sequel.

In this article the focus lies on the second of the at the beginning mentioned aspects – the investigation of a sample which is wetted once from the top and once from below. Therefore, the processes of infiltration and capillary rise are studied in the sequel. For a better understanding of the phenomena they are studied under simple conditions. A single soil type with uniform grain-size distribution (fine sand) is chosen and only initially dry material has been used. Moreover, investigations have been limited to one-dimensional, vertical flow.

2 Laboratory investigation of 1D capillary rise and infiltration of an initially dry fine-sand sample

The two processes infiltration and capillary rise are investigated in the simplest possible laboratory tests. Thus, one-dimensional tests on a single soil sample (fine sand) which is initially dry are undertaken. The physical properties of the sand are given in Table 1. The material is very uniform which means that the particles have nearly the same diameter.

One apparatus, an acrylic glass cylinder of 100 cm height and 9.19 cm inner diameter, is used to study both effects. Once the sample is watered from the bottom and once from above (see Fig. 1). The laboratory tests have been conducted by Moritz Riemenschneider as part of his master's thesis at the University of Duisburg-Essen [65]. Not only the duration of the movements in the sand sample was observed but also the type of transition zone between the wet and the dry part of the sample when the water moved up or down.

When water moves in dry sand, a wetting front is generated which shifts down for infiltration and up for capillary rise. This can be easily observed in the sequences of photos (Fig. 2) showing the temporal progress of the two processes capillary rise (a) and infiltration (b). The wetting front is the separating line between the wet and the dry region. It never moves completely uniformly because the water has to find its way through the pore space while the particles obstruct the flow. It is obvious that the flow is more consistent if the particles are of same size and are rather spherical. Depending on the conditions like uniformity and shape, size and distribution of the particles, initial water content and, if applicable, infiltration rate different shapes of the wetting front develop (see Fig. 3). Under certain circumstances the water rises or flows very irregularly through the pore space so that fingers, also called preferential pathways, build in which the water moves faster and contaminants are rapidly transported. Such a heterogeneous flow, also called fingering flow, develops as result of an instability at the wetting front [42]. Also the existence of coarser or denser layers with different permeabilities on top or beneath a soil layer may lead to the building of fingers [77]. In general fingering of a wetting front is discussed e.g. in [75].

As gets already obvious from Fig. 1 and possibly gets even clearer from the sequences in Fig. 2 the wetting front in the laboratory tests is almost plane – maybe for capillary rise a little bit more wavy than for infiltration. Only a very low level of fingering is visible. In both cases the wetting front is sharp. In the very homogeneous soil sample this behavior is not surprising even if in [42] it is pointed out that the initial water content is the most important factor for the formation of the wetting front. The authors describe that for infiltration processes also in homogeneous soils in initially very dry samples fingering appears while in initially wetter samples the wetting front is nearly plane. In the present tests the fingering despite the initially dry sample is very weakly pronounced. Also the expected influence at the walls of the cylinder has hardly occurred. Admittedly, by use of a special upper part which can be seen in Fig. 1 (c) it had been taken care in advance that the water could flow evenly into the sample.

Figure 2 shows chronological sequences of the two tests. The left hand side (a) shows that the capillary rise in the first minutes happens rather quickly and then approaches an asymptotic value. The test was carried out three times with a rather dense packing of the material and denoted by cr-dp-1 to cr-dp-3. In these tests the final value of the capillary rise was in the range from 32.6 to 33.4 cm. A fourth test with loose trickle-in, denoted by cr-lp, ensued a slightly smaller value of 28.1 cm. After a time of 120 minutes when the movements were mostly completed, the time intervals for measurements were enlarged. A measurement after 48 hours brought only slightly changed values. Therefore they are not shown in Figure 4 (a) in which the results for the capillary rise height of the four tests in dependence on time are given. The asymptotic value had nearly been reached after 120 minutes.

Figure 4 (b) shows the behavior of the infiltration depth in dependence on time. This test was performed two times on dense fine sand (i-dp-1 and i-dp-2) and once on loose fine sand (i-lp). In the first case it took 24 minutes to wet the sample, in the latter case only twenty minutes passed. As expected, the degree of compactness had the effect that the capillary rise was larger for dense packing while the infiltration was faster in the case

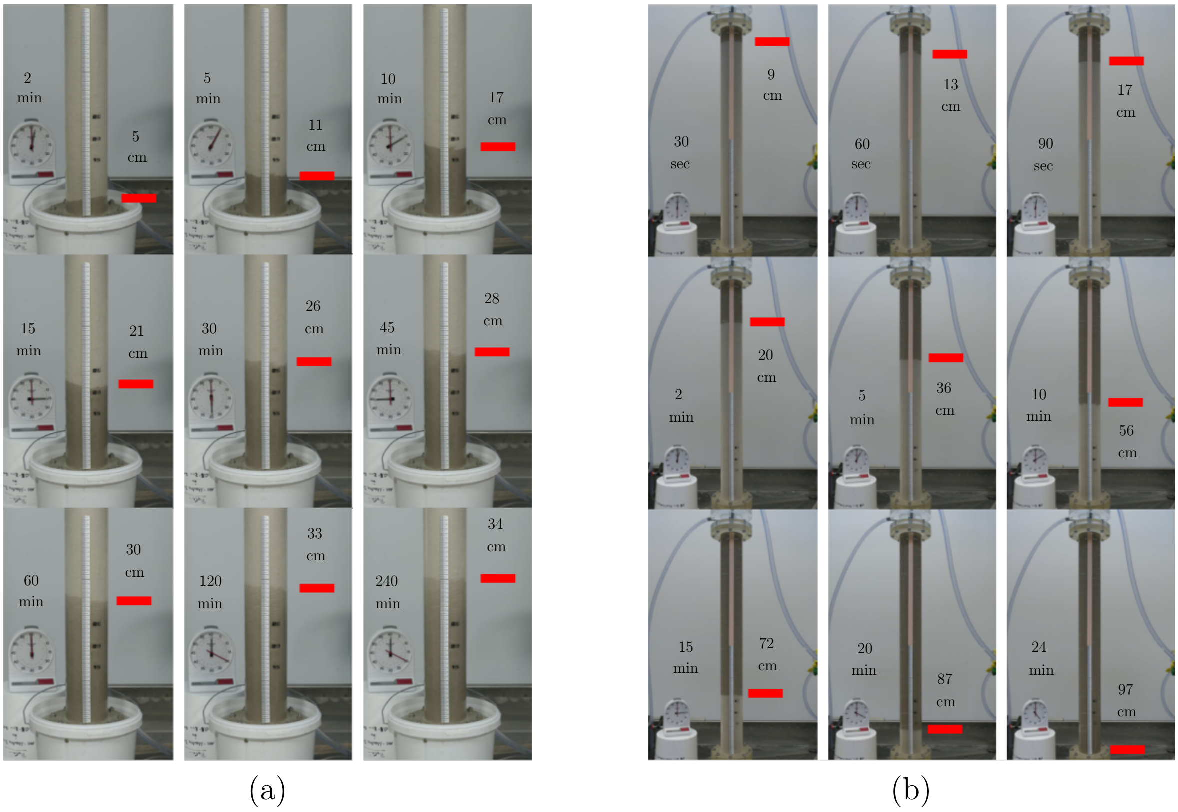


Figure 2: Chronological sequences of (a) capillary rise (test cr-dp-2 after 2, 5, 10, 15, 30, 45, 60, 120 and 240 minutes) and (b) infiltration using the same, initially dry, fine sand sample (test i-dp-2 after 30, 60, 90 seconds and 2, 5, 10, 15, 20 and 24 minutes); the red bar indicates the approximate distance in cm of the wetting front at the respective time; cr: capillary rise, i: infiltration, dp: dense packing, lp: loose packing; photos: Riemenschneider [65].

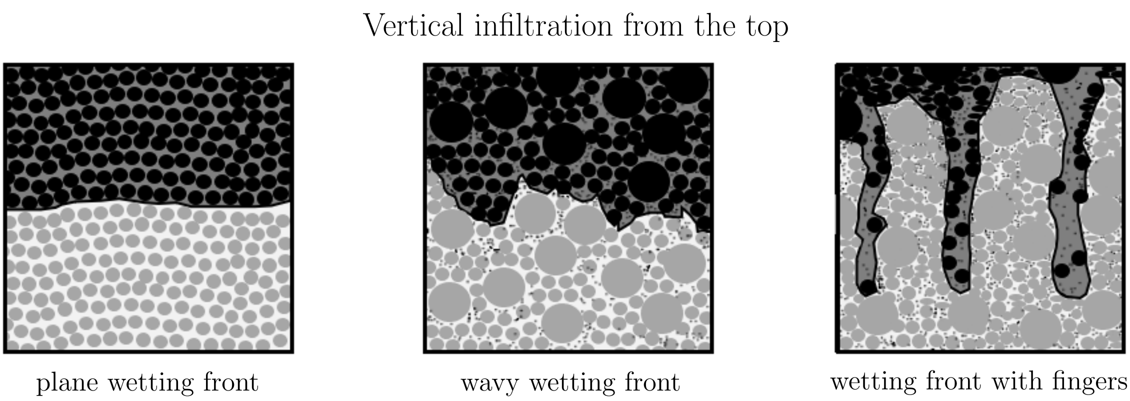
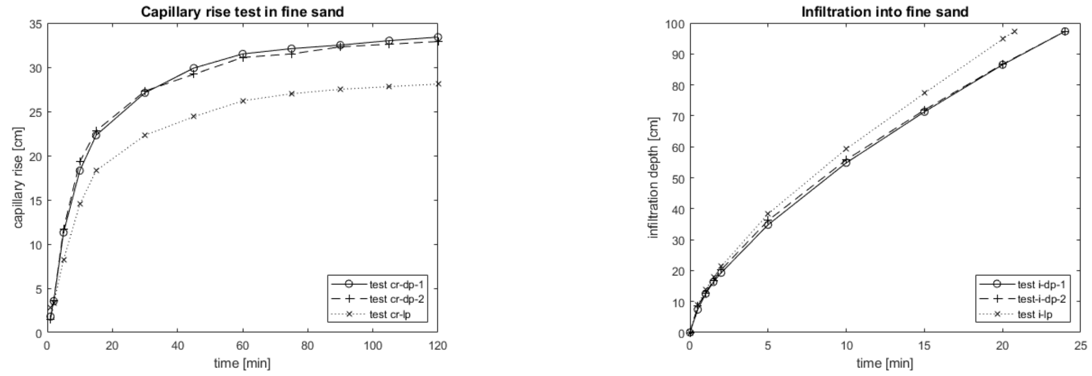
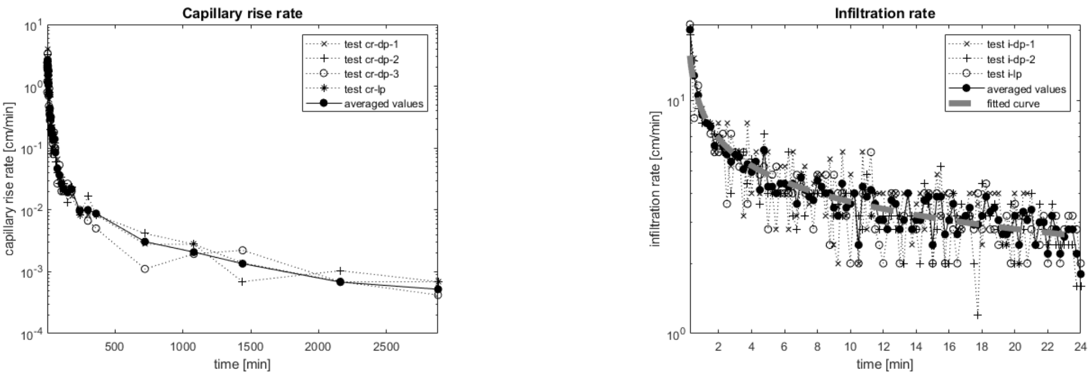


Figure 3: Possible types of wetting fronts for vertical infiltration into different soil types, dark gray: wet soil, light gray: dry soil.



(a) (b)

Figure 4: Capillary rise height (a) and infiltration depth (b) in dependence on time; dp denotes dense packing, lp loose packing.



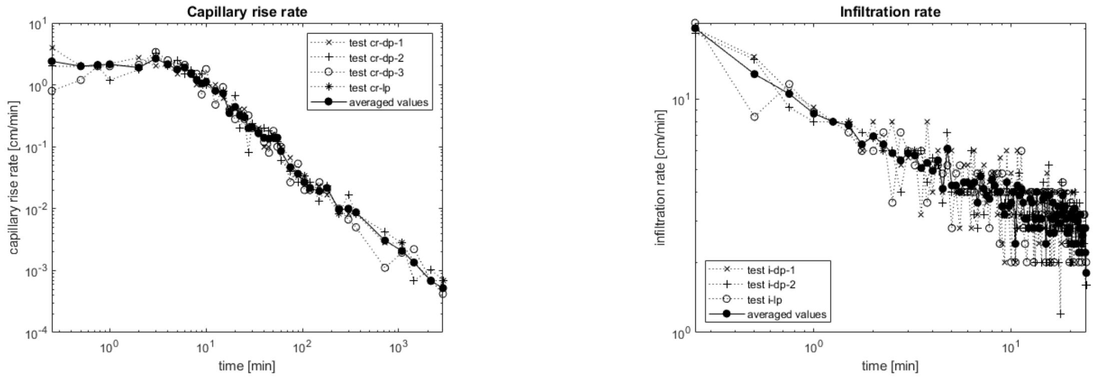
(a) (b)

Figure 5: Capillary rise rate (a) and infiltration rate (b) in dependence on time; dp denotes dense packing, lp loose packing.

of loose packing. In general, the infiltration process is faster than the capillary rise. The infiltration into the sand column of nearly one meter happens in around twenty minutes while the final capillary rise of around 30 cm takes two hours and more.

Interestingly, even if the curves in Figure 5 principally have the same form, if plotted in a log-log diagram, as done in Figure 6, they show differences. Namely, for the infiltration process the average rate can be represented by a single straight line. For capillary rise in the beginning of the process the rate is nearly constant and only after a few minutes the values also can be fitted by a line. This behavior will be discussed later and compared to attempts of explanation found in literature. In comparison to the infiltration rate the capillary rise rate changes very much. In the present tests the sample is initially dry and for very low values of the saturation the capillary pressure is immensely high. Admittedly, the behavior in this extremely dry region is practically not particularly relevant because in nature the underground mostly will contain a certain amount of water. It should be mentioned that the curve for capillary rise reminds very much the logarithmic presentation of the hydraulic conductivity over the pressure head.

Quantitatively the values for the capillary rise height can be compared to measurements which have been published already in 1947 in [45]. The authors carried out two different types of capillary rise measurements, namely capillarymeter tests and open soil filled tube



(a) (b)

Figure 6: Capillary rise rate (a) and infiltration rate (b) in dependence on time.

tests on eight different soil types. The latter method is comparable with the tests carried out here. Most comparable to the soil classes investigated in [45] is the sample of class 2, a sand with similar physical properties to the sample used here, for which the tube test resulted in a capillary rise height of 28.4 cm. The measurements for this soil type are also used later in Fig. 10. Even if the results for the certain soil classes are extremely different and the comparability is very limited, values of the same order are observed. Further capillary rise tests conducted in 2017 have been shown in [39]. Rising head column tests were performed using pure water in medium-grained poorly graded sand which also was initially dry. However, the focus did not lie on the rate of capillary rise and therefore the results of those tests are hardly to compare to the present ones.

Despite of the extremely simple structure of the infiltration tests also these are difficult to compare to existing results (for more advanced infiltration measurements visit e.g. [9]). This may be attributed on the one hand to the used type of soil which is very uniform in comparison to other infiltration tests which are often performed on natural ground because the penetration of rain is simulated. Another reason may lie in the simple experimental conditions. If compared to other infiltration tests it gets obvious that the tests pass very quickly. Comparable tests reported on a sand mixture in [68] revealed a wetting front movement of around 40 cm in 90 minutes. Interestingly, results of such simple tests on uniform, initially dry material as carried out here are very rare.

3 Theoretical investigation of 1D infiltration and capillary rise

Capillary rise and infiltration are certain forms of fluid flow. Even if both processes arise also in other materials such as concrete, masonry or other porous materials the interest in this paper lies on soils. An overview of early developments in soil physics can be found in [60]. More recent advances are summarized in the book [55]. Described are single-component approaches describing flow in soils. A more advanced possibility of theoretical description are multi-component models based on the mixture theory of fluids [74] which are at least then the opportune choice for the description of processes in soils, rocks and sediments if the relative movement of the pore fluid(s) to the solid component cannot be neglected. This is the case if the simplifying assumptions underlying simpler formulations as, e.g. small deformations or incompressibility, are not fulfilled. Also the multi-component

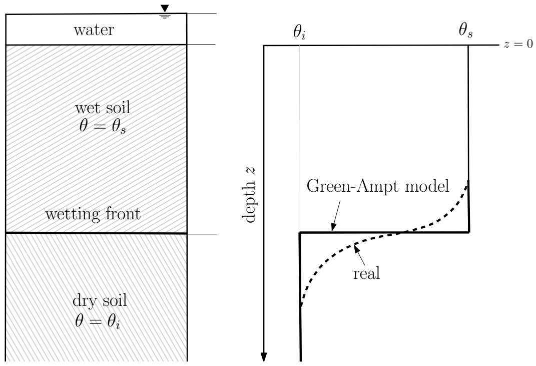


Figure 7: Soil water content profile for moving wetting front.

analysis of wave propagation in poroelastic media in which all constituents are considered to be deformable and have their own kinematics should be mentioned. It reveals wave types (e.g. the slow longitudinal wave in saturated poroelastic media) which obviously cannot be determined by single-component approaches. An overview of multi-component methods and several applications can be found in [3], [8], [25] or [26]. In the present article the focus lies on the simpler approaches.

3.1 Infiltration

It is intuitively clear that water that appears at a soil surface, whether through rainfall or other supplies, due to gravity tends to penetrate into the soil and to seep away. This process is called infiltration and is widely studied in the literature, see e.g. the reviews [10] or [13], the early approach on flow of air and water through soils [33], the study of vertical infiltration in dry soils [18], the investigation of infiltration equations for simple soil systems [24]or the very extensive research on infiltration by J.R. Philip which appeared 1957/58 in seven parts in Soil Science (here the later published compendium [59] is cited instead). This was discussed e.g. in [56] or the comparison of some numerical simulation models for 1D infiltration [36]. If the soil is bounded by a water table the process is called ponded infiltration which is studied e.g. in [66].

Many empirically derived expressions for the infiltration rate or the cumulative amount of infiltration are in use. As pointed out in [41] they share the common feature that the infiltration rate is highest when water first enters the soil and decreases with time as the wetting front moves away from the surface. As can be seen from the test results shown in Fig. 5 (b) this behavior is confirmed by the above shown laboratory tests. For the theoretical description of one-dimensional infiltration first some exponential laws have been proposed (e.g. Lewis equation, Kostiakov equation, Horton equation) which all have the problem to specify the constant parameters appearing in them and which mostly only fit experimental results. In contrast, the Green-Ampt model proposed in 1911 [33], which sometimes is still used today, is based on physical considerations which are summarized below.

Green-Ampt model

During an infiltration process with a ponding water layer the surface is held at constant pressure head denoted by h_s. The associated water content of the saturated, wet soil is θ_s

and the water enters the soil behind a sharply defined wetting front moving downwards with time. This is exactly what has been observed in the series of figures in Fig. 2. In the dry part of the soil the water content has the irreducible value θ_i. In the Green-Ampt model it is assumed that the change in water content at the wetting front is discontinuous (see Fig. 7). Further assumptions have been made, namely that the soil in the wet part has constant properties and that the pressure head at the moving front is constant and equal to h_f.

The process of vertical infiltration is considered. At the soil surface $z = 0$ the hydraulic head is equal to the pressure head $H_s = h_s$. At the front, $z = d$ is $H_d = d + h_f$. Thus, Darcy's law across the wet region is (see Eq. (A.1))

$$q = -K_{GA}\frac{H_d - H_s}{d} = \frac{K_{GA}}{d}\left(\Delta h + d\right), \tag{1}$$

with $\Delta h = h_s - h_f > 0$. Then,

$$q = \Delta\theta \frac{\mathrm{d}d}{\mathrm{d}t} = \frac{K_{GA}}{d}\left(\Delta h + d\right). \tag{2}$$

Rearranged and integrated this is

$$\int_0^d \frac{d\,\mathrm{d}d}{\Delta h + d} = \frac{K_{GA}}{\Delta\theta}\int_0^t \mathrm{d}t. \tag{3}$$

Performing the integrals yields

$$d - \Delta h \ln\left(1 + \frac{d}{\Delta h}\right) = \frac{K_{GA}t}{\Delta\theta}. \tag{4}$$

According to [41], for short times, soon after infiltration begins, this leads to a cumulative infiltration

$$I = \Delta\theta\sqrt{2K_{GA}\frac{\Delta h}{\Delta\theta}t}, \tag{5}$$

and, accordingly, an infiltration rate of

$$\mathrm{i} = \Delta\theta\sqrt{K_{GA}\tfrac{\Delta h}{\Delta\theta}/2t}. \tag{6}$$

At very large times results $\mathrm{i} \approx K_{GA}$.

To apply the Green-Ampt model the effective hydraulic conductivity, the pressure head at the wetting front, the porosity and the initial water content have to be known. According to [63] the effective hydraulic conductivity K_{GA} has to be determined by a special formula. However, often it is stated that for simplicity it can be assumed to be half of the saturated hydraulic conductivity K.

<u>Comparison to laboratory tests</u> Since the hydraulic head is not measured in the above mentioned laboratory experiments a comparison of the test results to the Green-Ampt model is not possible. Only the estimate for very long times can be compared: then $\mathrm{i} \approx K_{GA} \approx \frac{1}{2}K$ which is for dense packing $\frac{1}{2} \cdot 1.116$ cm/min $= 0.558$ cm/min. The fitted curve in Figure 5 reveals a clearly higher value of nearly 2 cm/min. It has to be mentioned that the estimated value is completely independent of the ponding height which is an unrealistic assumption.

Philip model

Philip was one of the first researchers who tried to solve the nonlinear Richards equation for a special problem, namely the infiltration into an infinitely deep homogeneous porous medium at uniform initial water content. Just like the Green-Ampt model the vertical infiltration solution of Philip appears to have two separate expressions for short and long times [41]. The short-time solution is expressed as an infinite series in powers of the square root of time

$$I = St^{1/2} + A_1 t + A_2 t^{3/2} + ..., \tag{7}$$

where S is the sorptivity. This is a quantity Philip introduces in his numerous works on infiltration in the following way [58]: "A new physical property of porous media, sorptivity, is proposed. In some ways akin to permeability, this is essentially a measure of the capacity of the medium to absorb or desorb liquid by capillarity ...". Both S and the constant parameters A_n depend on the soil properties as well as boundary and initial water contents θ_0 and θ_i. Generally, only the first two terms are considered so that the Philip model for short times is approximated by

$$I = St^{1/2} + At, \tag{8}$$

for the cumulative infiltration and the corresponding infiltration rate

$$\mathrm{i} = \frac{1}{2} St^{-1/2} + A. \tag{9}$$

For long times the infiltration rate approaches a constant equal to the value $K(\theta_0)$, the unsaturated hydraulic conductivity at the surface water content. Thus, $\mathrm{i} \to K(\theta_0)$.
Comparison to laboratory tests Apart from the suggestion to take only half of the hydraulic conductivity value this is equivalent to the Green-Ampt approach because at the surface where the water is ponding the medium is saturated.

Numerical solution of the Richards equation

In the book [16] the predictor-corrector method to solve the Richards equation (A.8) is described and on the webpage [15] the matlab code developed to this solution method is downloadable. Using these files one is able to calculate and plot, for example, soil moisture profiles. Since in the laboratory tests only the penetration depth of the wetting front and not the change of the water content itself has been measured, such numerical results cannot be compared to the test results. However, in [36] for sand additionally to the soil moisture profiles also the cumulative infiltration and the infiltration rate are calculated and compared with Philip's solution. This is reproduced in Figure 8.
Comparison to laboratory tests Compared to Figure 4 (b) and the fitted curve in Figure 5 (b) it gets obvious that the qualitative behavior is the same.

3.2 Capillary rise

Capillary rise is also very frequently but maybe not as extensively theoretically investigated as infiltration.

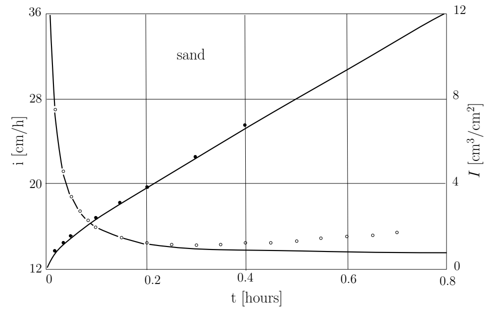


Figure 8: Calculated cumulative infiltration I [cm^3/cm^2] (filled circles) and infiltration rate i [cm/h] (open circles) in sand (after [36]).

Early approaches

There are two helpful overviews on the topic [67], [47]. The latter of 1964 immediately reveals that until this date mostly formulae have been indicated which reproduce experimental results. They are based on the Young-Laplace equation from which the ultimate height of capillary rise h_c in a capillary tube can be described by

$$h_c = \frac{2\gamma}{\rho g r} \cdot \cos\theta, \tag{10}$$

where γ denotes the surface tension, ρ the mass density of the fluid, g the earth acceleration, r the radius of the capillary tube and θ the wetting angle between fluid and pipe wall. As examples the formulae of Zunker (1924), Kozeny (1927), Carman (1941) and Terzaghi (1943) are mentioned.

Comparison to laboratory tests Carman also measured the capillary rise height in his work [23] and obtained in fine sand a height of 30 cm in 20.4 minutes which is quantitatively consistent with the present results.

Terzaghi's work and enhancement

Because Terzaghi's approach [72] according to [48] significantly overpredicts the rate of capillary rise the authors proposed an enhanced version. Terzaghi assumed that the hydraulic gradient i is responsible for capillary rise and can be approximated by

$$i = \frac{h_c - z}{z}, \tag{11}$$

where z is the distance measured positively upward from the elevation of the water table (see Fig. 9). As pointed out in [48], physically, h_c represents the drop in pressure head across the air-water interface at the wetting front in the soil pores (compare to Figure 7). A second assumption is the validity of Darcy's law in the form (see Eq. (A.2) in the appendix) where according to the present sign convention the minus sign is missing. According to Terzaghi [72] the rate $\mathrm{d}z/\mathrm{d}t$ at which the upper boundary of the saturated zone rises in vertical direction is identical with the seepage velocity v_s. This is the vertical component of the average velocity of the water in the capillary channels

$$v_s = \frac{v}{n} = \frac{\mathrm{d}z}{\mathrm{d}t}, \tag{12}$$

where n is the porosity of the soil. Assuming zero capillary rise at zero time, Terzaghi arrived at the following solution describing the location of the wetting front z as an implicit function of the time

$$t = \frac{nh_c}{k}\left(\ln\frac{h_c}{h_c - z} - \frac{z}{h_c}\right). \tag{13}$$

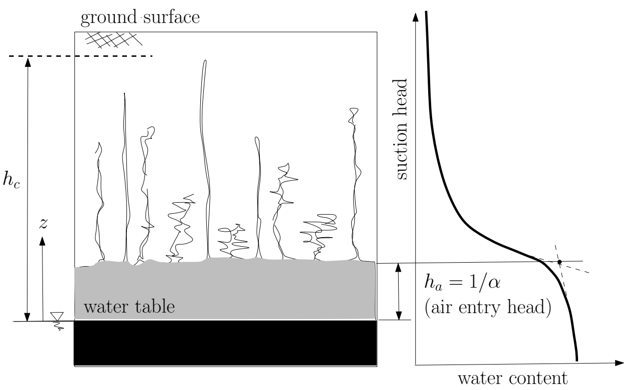


Figure 9: Sketch of the situation in an unsaturated soil and comparison to the respective capillary pressure or suction head for a certain water content (after [48]).

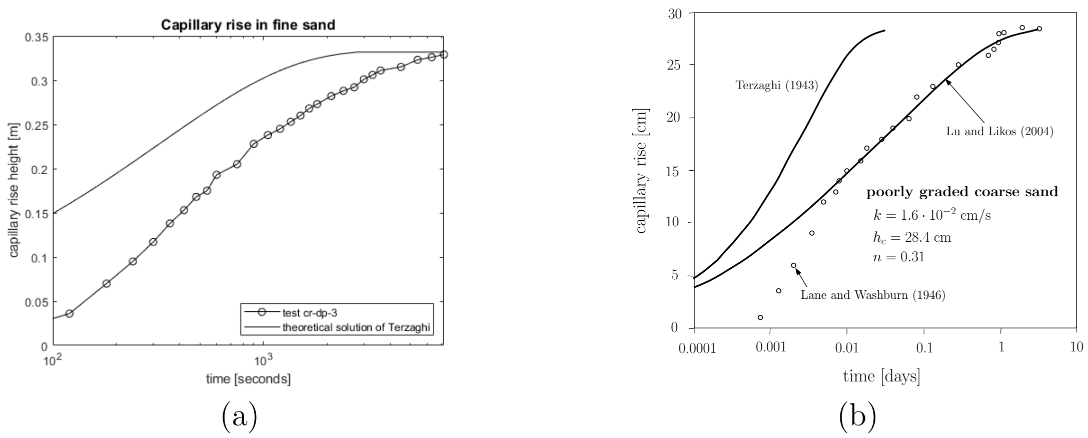


Figure 10: (a) Comparison test results and theoretical solution of Terzaghi, (b) comparison solutions of Terzaghi and Lu & Likos for a poorly graded coarse sand (after [48]).

Comparison to laboratory tests Although realizing that the assumption is unrealistic Terzaghi used a constant value for the hydraulic conductivity. However, e.g. in [64] it is pointed out that it decreases dramatically if the water saturation decreases (already a small air content has huge impact). Thus, Terzaghi's formula overpredicts the rate of capillary rise as can be seen in Figure 10 (a). The figure shows a comparison of present test results to Terzaghi's theoretical solution using the material parameters for the dense packing of the fine sand.

Since that time several formulae using either exponential, power or series functions have been introduced to account for the drastic reduction of hydraulic conductivity with increasing suction. Using an exponential function introduced by Gardner in 1958 Terzaghi's approach in [48] in 2004 has been modified

$$\frac{\mathrm{d}z}{\mathrm{d}t} = \frac{k}{n} \exp\left(-\alpha z\right) \frac{h_c - z}{z}, \tag{14}$$

and a solution has been found. It contains the constant saturated hydraulic conductivity k multiplied by the exponential function with parameter α representing the rate of decrease in hydraulic conductivity with decreasing suction head. This is proportional to the width of the pore size distribution of the soil. Figure 10 (b) shows a comparison of Terzaghi's original solution and the modified solution by Lu and Likos for a poorly graded coarse sand.

Washburn equation

Above, the experimental study on the height and rate of capillary rise on eight dried soil classes was mentioned [45]. The work of Washburn resulted additionally in a theoretical description of the phenomenon which is widely used until today [78]. He considered cylindrical capillary tubes of certain radius and asked the question how much liquid will enter the porous medium at a certain time t. For a constant driving pressure he found the relation

$$V = k' \sqrt{\frac{\gamma}{\eta}} \sqrt{t}, \tag{15}$$

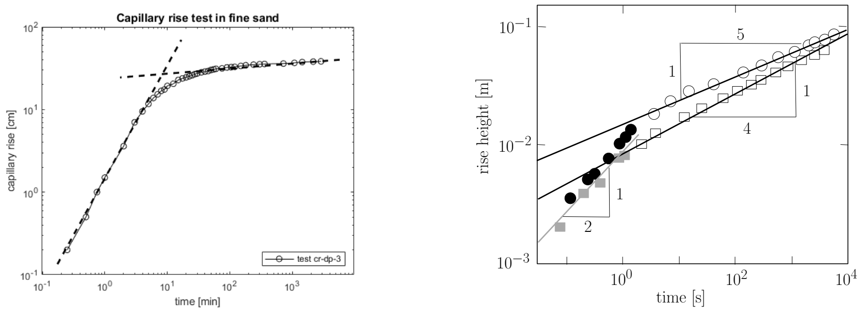
where the parameter k' is independent of the nature of the liquid. The degree of penetration V is proportional to the square root of the time of soaking and to the square root of the ratio of the surface tension γ and the viscosity η. Washburn himself restricted the applicability of this relation. He pointed out that it would not hold if the pores of the body cannot be taken as equivalent in their behavior to cylindrical pores, if the cross-section of a pore changes with its length and especially if the pore contains an enlargement or ends in a pocket. Furthermore, these equations would in all probability not apply to the filling of pores with diameters approaching the molecular diameter of the liquid. Nevertheless, Eq. (15) has proven to show understandable results at least for a part of the duration of the process of capillary rise.

Comparison to laboratory tests If the capillary rise height of the tests shown above is plotted in log-log form, as done in Figure 11, it gets obvious that both the capillary rise for small times and the height for large times can be aligned with a straight line. This behavior could also be observed in [34] in which the capillary rise in cellulose sponges using different liquids has been investigated. It is shown very vividly that the sponge swells if water or aqueous liquids are used while for turpentine the liquid only rises in the sponge. Irrespective of the type of liquid it can be observed that in the early stages the rise height grows like $\sqrt{t}$, while in the late stages the rise height followed another, fluid dependent rule, in this case $t^{1/5}$ for water and $t^{1/4}$ for turpentine. It is stated that the power law of the height changes when the rise height reaches Jurin's height of macropores which is defined by

$$h_J = \frac{\gamma}{\rho g r}, \tag{16}$$

including the surface tension γ, the mass density of the liquid ρ, earth acceleration g and the radius r of the capillary through which the liquid rises.

Also in the here shown tests of water rising in sand such regions can be observed (see Fig. 11): one region for small times and one for later times. It is possible that in the



(a) (b)

Figure 11: (a) Capillary rise height in log-log presentation of the test for water in fine sand, (b) rise height for water and turpentine in a cellulose sponge (after [34]).

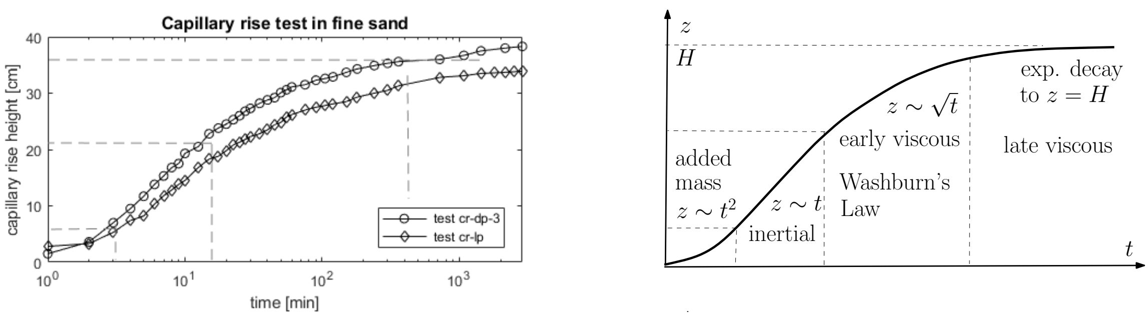
first region the rise height also grows like $\sqrt{t}$, in the latter region the slope is significantly lower and also there seems to be a more pronounced transition zone between the two regions. This implies that the different regions have to be attributed to different trigger mechanisms acting in a certain period of the process.

Identification of several time regimes by comparison to literature results

In Fig. 12 (b) a diagram from [21] is shown in which several regimes of a capillary rise process are shown. At different stages of the process certain forces or mechanisms are acting. The conservation of momentum is noted there as follows

$$\Big(\underbrace{m}_{\text{inertia}} + \underbrace{m_a}_{\text{added mass}}\Big)\ddot{z} = \underbrace{2\pi r\gamma\cos\theta}_{\text{capillary force}} - \underbrace{mg}_{\text{weight}} - \underbrace{\pi r^2 \frac{1}{2}\rho\dot{z}^2}_{\text{dynamic pressure}} - \underbrace{2\pi rz\cdot\tau_v}_{\text{viscous force}}, \tag{17}$$

with $m = \pi r^2 z\rho$. In the diagram certain periods of time are attributed to the certain mechanisms. And if we compare the diagram on the right hand side of Fig. 12 (b) to this on the left hand side (a) where the capillary rise height of the present tests is plotted over the time in logarithmic presentation then it gets obvious that also in the present case such time regimes can be recognized. It means that in the beginning of the process there is a very small time region in which added mass effects dominate (for explanation see [21]), a period follows in which inertial effects are crucial, then an early viscous period in which Washburn's law holds is observed and, finally, the curve experiences an exponential decay. In these time periods the capillary rise height changes its dependence on time from t^2 over t and $\sqrt{t}$ to constant. At the end an asymptotic value of the capillary rise height is reached. In [32] and [37] the authors come to similar results but focus on the three regions driven by inertia, viscosity and gravity.
The dynamics of capillary rise has recently been studied also e.g. in [22], [30], [31], [50], [52], [53], [69] or [80]. A certain analogy with Philip's approach of infiltration processes (compare Eq. (7)) is also noticeable.



(a) (b)

Figure 12: (a) Capillary rise height in log presentation for time of the test for water in fine sand, (b) scaling regimes of capillary rise (after [21]).

4 Conclusions and outlook

For the presented tests and considerations in all respects the simplest possible way has been chosen. Capillary rise and infiltration have been experimentally studied under simple conditions on one single soil type with uniform particle size distribution. The problem has been studied in 1D. This does, of course, not represent the initially mentioned example in a flower pot. Nevertheless, it could be shown that the results of the tests are consistent with theoretical approaches. The agreement of quantitative results was not always given but qualitatively the results agree with other observations even if the comparability was limited due to other materials, processes or other conditions.

The simplicity had been chosen to get a better insight into the processes themselves. In future investigations, for example, the influence of the soil type can be tested. Thus, other soil types with different grain sizes and shapes, a lower degree of uniformity or layers of different soil types could be investigated. Of course, for infiltration is of interest how the wetting front spreads in two or even three dimensions.

One important point which has been mentioned several times in this article is the initial moisture content of the sample. Here, only initially dry samples have been studied. But, as presented, the initial moisture content has an immense impact on the penetration ability of a liquid into a porous medium. Especially in view of the current climate situation with an increasing number of droughts on the one hand side and heavy rains on the other side an investigation of the influence of the initial conditions is beneficial.

If, indeed, the example of the flower pot mentioned in the beginning is studied, of course, also aspects like the root system of the plants, the repellency of the soil, swelling and shrinking of the material and other ecological issues have to been taken into account.

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A Basic notions and description of flow processes

Darcy's law

Fluid flow in saturated soils or generally porous media is described by Darcy's law. It can be expressed in some ways. One of them is (in one-dimensional form)

$$q = \frac{Q}{A} = -K\nabla H, \tag{A.1}$$

where $q\ \left[\frac{\mathrm{m}}{\mathrm{s}}\right]$ is the flow velocity, also called specific discharge. It is equal to the quotient of the flow rate $Q\ \left[\frac{\mathrm{m}^3}{\mathrm{s}}\right]$ and the flow cross section area $A\ \left[\mathrm{m}^2\right]$, K is the hydraulic conductivity[2] and H is the hydraulic

[2]The notation of permeability parameters is inhomogeneous and at least for Europeans a little bit confusing since in European standards different notations are common than mostly used in literature. Here, as common in the geophysics and hydrology literature (see e.g. [11]), the hydraulic conductivity is denoted by K and has the unit $\left[\frac{\mathrm{m}}{\mathrm{s}}\right]$ or $\left[\frac{\text{length}}{\text{time}}\right]$ and $k = K\frac{\eta}{\rho g}$ $\left[\mathrm{m}^2\right]$ or $\left[\text{length}^2\right]$ is the permeability parameter. Therein, η is the viscosity of the fluid, ρ is the mass density of the fluid and g is the earth acceleration. In contrast, in DIN EN ISO 17892-11 (Geotechnical investigation and testing – Laboratory testing of soil – Part 11: Permeability tests) the permeability coefficient k has units $\left[\frac{\mathrm{m}}{\mathrm{s}}\right]$ or $\left[\frac{\text{length}}{\text{time}}\right]$ and this coefficient, often also denoted by k_f, is introduced in almost all textbooks on soil mechanics (e.g [44], [46] or the classical book of Terzaghi [72]) and used in soil mechanics education.

or piezometric head. The latter is also called potential ϕ (similarly to the permeability parameters mainly in the European literature on soil mechanics) and is according to Bernoulli's equation approximately (the term containing the square of the velocity has been neglected) the sum of the gravitational head (or elevation) z and the pressure head $h = \frac{p}{\rho g}$, where p is the pore pressure, ρ is the density of the fluid and g the earth acceleration (gravity). A head is defined as work per unit weight of water and has units of length. Another definition of the potential would be to refer the work to the unit volume of water, then it has units of pressure. Often these notions are used interchangeably so that e.g. the pressure head and the matric potential mean the same.

Darcy's law only applies under simplifying assumptions as, for example, laminar flows and an incompressible fluid with constant temperature. If the notation of the footnote is used, another form of Darcy's law arises, namely

$$v = -k_f \cdot i. \tag{A.2}$$

Here, v is the filter velocity, i.e. a fictitious velocity which would appear if the fluid could flow unhindered through the body. The filter velocity does not take into account that a porous medium exists of solids and the pore space and is defined as if the fluid could flow both through the pore space and the particles. This definition gets also obvious from (A.1) because A is the whole flow cross section area and not only the surface area of the pore space. Moreover, (A.2) contains the hydraulic gradient i which is equivalent to ∇H in (A.1). It governs the flow since it is the change of the hydraulic head over a distance along the flow path. The right hand side of both equations is negative because principally gradients describe the increase from a small to large value. However, the flow of a fluid, which is described here, happens from a larger to a smaller potential or water level, respectively.

From Darcy's law it gets obvious that the flow through a medium is proportional to the hydraulic gradient. This also follows if Poiseuille flow through cylindrical tubes is considered. Such a laminar flow and a geometry of pores with constant cross-section in soils do not exist. Anyhow, Poiseuille's approach which can be derived under certain limiting conditions from the Navier-Stokes equations shows that the radius of the pores and also the viscosity of the fluid play an important role. According to Poiseuille's law the flow rate $q\ \left[\frac{\mathrm{m}}{\mathrm{s}}\right]$, above also called flow velocity or specific discharge, has the form

$$q = \frac{r^2}{8\eta} i. \tag{A.3}$$

It can be seen quickly that also in a geometrically less homogeneous geometry the radius of the flow channels r and the viscosity of the pore fluid η are decisive quantities. Moreover, a comparison of equations (A.2) and (A.3) shows that these parameters control the permeability in a porous medium.

Buckingham's extension of Darcy's law

The pore space of natural soils in general will be occupied by both water and air. Possibly, either water- or air-saturated regions exist but there will appear also regions in which two (or more) immiscible fluids are present. If water and air fill the pores the degree of saturation of each of the two pore fluids can be defined as the ratio of the volume of the specific fluid to the entire volume of the voids, i.e. $S_W = \frac{V_W}{V_V}$ for the water saturation and $S_A = \frac{V_A}{V_V}$ for the air saturation. In hydrology commonly the soil water content is used instead. Then the ratio of the water volume is referred to the entire volume of the soil sample, i.e. $\theta = \frac{V_W}{V} = \frac{V_V}{V} S_W = n S_W$, where n is the porosity, i.e. the ratio of the void volume over the entire volume. For unsaturated (or also called partially saturated) flows, in which two or more immiscible pore fluids fill the pore space, Buckingham modified Darcy's law (A.1). Under the assumptions that the flow is isomotic and isothermal (effects of salt and temperature variations are neglected) he took into account that the hydraulic conductivity and the hydraulic head are functions of S_W or θ, respectively.

Recognizing the above mentioned dependencies, Eq. (A.1) for one-dimensional, vertical flow in unsaturated soils becomes

$$q = -K(\theta) \left[\frac{\partial h(\theta)}{\partial z} + 1 \right]. \tag{A.4}$$

The interested reader in Buckingham's works about flow in unsaturated soils is referred to e.g. [20], [54] or [63].

Richards equation

Darcy's law (A.1) in general, one-dimensional form,

$$q_z = -K\frac{\partial H}{\partial z}, \qquad \text{with} \qquad H = h + z, \tag{A.5}$$

combined with the mass conservation law (continuity equation) which can be derived comparing the in- and outflow across an increment of a cuboid (see e.g. [73]),

$$\frac{d\theta}{dt} = -\frac{\partial q_z}{\partial z}, \tag{A.6}$$

results in

$$\frac{d\theta}{dt} = \frac{\partial}{\partial z}\left[K\frac{\partial H}{\partial z}\right]. \tag{A.7}$$

If the soil is unsaturated and K depends on θ the Richards equation can be specified

$$\begin{aligned} \frac{d\theta}{dt} &= \frac{\partial}{\partial z}\left[K(\theta)\frac{\partial H}{\partial z}\right] = \frac{\partial}{\partial z}\left[K(\theta)\left(\frac{\partial h}{\partial z} + \frac{\partial z}{\partial z}\right)\right] \\ &= \frac{\partial}{\partial z}\left[K(\theta)\left(\frac{\partial h}{\partial z} + 1\right)\right] = \frac{\partial}{\partial z}\left[K(\theta)\frac{\partial h}{\partial z}\right] + \frac{\partial K(\theta)}{\partial z}. \end{aligned} \tag{A.8}$$

If further is taken into account that a change in water content is accompanied by a change of the pressure head, i.e. $\frac{\partial\theta}{\partial t}=\frac{\partial h}{\partial t}\frac{d\theta}{dh}$, one obtains for one-dimensional flow in z-direction the so-called Fokker-Planck equation (see [11])

$$\frac{\partial h}{\partial t}\frac{d\theta}{dh} + \frac{\partial}{\partial z}\left[K(\theta)\frac{\partial h}{\partial z} - K(\theta)\right] = 0. \tag{A.9}$$

This type of partial differential equation has first been derived to describe Brownian motion and is used in statistical physics of non-equilibrium states. Underlying is the classification of diffuse processes into a sub-class of stochastic processes. Since the equations describing flows in unsaturated soils can be identified with this equation type also stochastic approaches appeared in the literature, e.g. [79]. The authors of [70] argue that the heterogeneity of a porous medium with irregular pores causes the flow to generate random fluctuations in the velocity field resulting in a stochastic component of the flow dynamics.

To arrive at equation (A.9) it is considered that the capillary pressure p_c in the mixture of the two immiscible fluids water and air is defined as the difference of the pressures of the two fluids (see [11] or [3])

$$p_c = p_A - p_W = p_{\text{air}} - p_{\text{water}}.$$

From (A.5) it is known that the piezometric head is defined as $H = h+z$. For flow of water in a saturated medium this means $H = \frac{p_W}{\rho_W g} + z$ at every point of the flow domain (see Bernoulli equation introduced above). For unsaturated flow and atmospheric air pressure, $p_{\text{air}} = 0$, the capillary pressure head is $h_c = \frac{p_c}{\rho_W g}$ and the capillary head

$$\phi_c = H_c = \frac{p_W}{\rho_W g} + z = -\frac{p_c}{\rho_W g} + z = -h_c + z \quad \text{for} \quad p_c > 0.$$

As pointed out in [11] and considered in the model for unsaturated porous media [4] it is important that the capillary pressure is a phenomenon on the microscopic level while in models one describes the averaged quantity on the macroscopic scale. Principally, one could set up models also on the microscopic level but then one would have to know boundary conditions on the surface of each soil particle which is not the case. Therefore, mostly models are formulated at the macroscopic scale so that the boundary conditions are only needed at the boundaries of the Representative Elementary Volume (REV).

Provided the flow is steady, the components are incompressible and the permeability is isotropic, a situation which is mostly assumed for flows in saturated porous media, equation (A.8) can be reduced to the Laplace equation in 1D

$$\frac{\partial^2 H}{\partial z^2} = \Delta H = 0. \tag{A.10}$$

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